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THE IMPORTANCE OF FLUENT COMPONENT SKILLS IN
MATHEMATICAL COMPREHENSION

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree
of Master of Arts

in

The Department of Psychology

by
Chisato Komatsu
B.A., University of Texas at Austin, 2001
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ABSTRACT

The primary question to be addressed by the present study was whether fluency on component skills is important in the development of overall competency in mathematics. Reading fluency has served as an excellent predictor of one's reading comprehension. However, few studies have investigated whether the fluency on component skills is essential in the development of overall competency in mathematics. In fact, there has been a push for instructional strategies to deemphasize the importance component skills. In the current study, 140 students in second- through fourth- grade classrooms from general education participated. Each student took three curriculum-based measurement probes (a single-skill mathematical computation probe, multiple-skill mathematical computation probes and maze reading passages), a mathematical reasoning probe, the Big Ideas probe and the Stanford Diagnostic Mathematics Test, Fourth Edition. Results of the six assessments were compared to determine if a fluency in component skills will adequately predict students' mathematical comprehension. Results demonstrated that the fluent component skills are in fact highly related to students' mathematics comprehension.

INTRODUCTION

The purpose of school is to help children to develop competence in various areas of academic functioning. There is little debate about this general goal. However, the means by which to achieve the goal of assisting students has stirred considerable discussion in literacy and in mathematics. In fact, there is even disagreement about the meaning of the term competence. The focus of this study is to examine mathematical competence or “comprehension” and its relationship to mathematics computational fluency. This preliminary study is designed to examine this relationship in order to begin to better understand mechanisms for improving mathematical competency.

The study of mathematics is particularly important in light of studies such as the Trends in International Math and Science Study (TIMSS), which reported that the fourth and eighth grade students in this country are performing below average in mathematics achievement (NCES, n.d.). In 1999, U.S. eighth-grade students ranked nineteenth in mathematics out of 34 nations. This is markedly lower than some of the higher achieving countries, which included Singapore, Korea, Chinese Taipei, Hong Kong, and Japan. Although some believe that students in other high achieving countries spend more time engaged in the study of mathematics, TIMSS conducted ethnographic studies on teachers and students in the United States and Japan and found that American students spend more time studying mathematics than Japanese students in school (American Federation of Teachers, n.d.).

Based on the TIMSS results, Koretz, McCaffrey, and Sullivan (2001) compared the variability of performance in the United States with that of several other countries. The researchers further investigated the variation distributed within and between

classrooms. Compared to that of Japanese and Korean schools, in the United States, much of the variance lies between classrooms rather than within a classroom (Koretz et al, 2001). These findings remind us of the influence of instruction on students' performance and suggest the locus of the problem may lie with instructional practices.

There are wide differences in the type of instructional practices a teacher uses. Obviously, the curriculum chosen by a school district plays a large role in the types of activities and objectives that a teacher may select. Importantly, high stakes testing influences instruction because many teachers attempt to align instruction with the objectives assessed by the test so as to improve student performance. Prior to entering a classroom, teachers are influenced by the type of training they receive at colleges and universities. All of these separate forces on the teacher, including the curricular materials from publishers, the construction of high stakes tests, and the content of university teacher preparation programs, have increasingly been influenced by a constructivist view that prescribes what needs to be learned, how it should be taught, and how to test for it.

A central assumption of the constructivist approach is the notion that students will become more proficient at mathematics if they discover the rules and methods of mathematics for themselves rather than via direct instruction from the teacher. (Mathematically correct, n.d., para. 2). Led by groups such as the National Council of Teachers of Mathematics (NCTM), there has been a push for instructional strategies that honor individual students' thinking and reasoning and encourage students to develop skills to solve problems (NCTM, 2000, p. 75). Although the constructivist approach is quite prominent among mathematics educators (Kamii, 1991), it is not the only

instructional methodology being promulgate. An alternative approach to constructivist theory derives out of learning theory (Binder, 1993). The practices advocated from this perspective, while quite different from the instructional practices used within the constructivist model, have in sight a similar outcome: assisting children to become competent in mathematics. Learning theory, which is also referred to as behavioral theory, posits that competency is arrived at not by having students construct their own schemas but by breaking complex skills into component parts and then teaching these skills in a linear sequence from least to most difficult. The constructivists have criticized (Kamii, 1991) this approach because they suggest teaching skills in isolation divorces the skills from meaning. How does a child, for example, come to understand that the symbol “3” represents three physical objects when the child is mindlessly counting the number or adding it to other numbers without considering for a moment what the symbol *means*.

A major point of divergence between the two views centers on the value of component skills. From a behaviorist perspective, component skills are the essential building blocks of all higher order skills and, in fact, it is difficult, if not impossible to become competent at higher level skills without first becoming proficient at lower order component skills. For example, they would argue that becoming proficient at algebra is difficult if you are not proficient at factoring, which in turn requires proficiency in division, and so on. Proficient for the behaviorist means that a skill has been practiced to a very high level of fluency such that the skill can be performed almost automatically (Binder, 2003). Constructivists have labeled the rote practice of component skills as “drill and kill” because they believe repeated drill and practice kills interest in mathematics (Kamii, 1991; Kamii, Lewis, & Livingstone, 1993).

The primary question to be addressed by the present study is whether fluency on component skills is important in the development of overall competency in mathematics. Behavioral theory would predict fluency on component skills is important whereas constructivist theory would predict that at best fluency on component skills is irrelevant to attaining competency in higher order skills. The question is important because it has relevance to mathematics instruction in that these two theoretical frameworks of mathematics instruction lead to differing notions about what type of practices lead to *competence* and even what competence is. In the next section, some background is provided about the basic underpinnings of the behavioral versus constructivist models.

Behaviorism and Constructivism

Behaviorism and Constructivism are two basic approaches that influence current educational practice. Behaviorism is an approach that focuses on effective teaching to establish fluency in basic component skills and their underlying tool elements in order to attain competency (Johnson, 1991; Johnson & Layng, 1993). Constructivism is based on Piaget's theory and holds that knowledge is something to be constructed, rather than acquired. The Student's task is creating and coordinating relationships. In the following sections, major tenets of the Behavioral and Constructivist theories will be delineated.

Constructivist Approach

The Constructivist approach is based on the idea that children construct knowledge by creating and coordinating relationships (Kamii, 1991). In order to ultimately construct strategies for solving mathematical problems, the approach emphasizes three major components: Discovery Learning, Whole Math, and Cooperative Learning.

Discovery Learning. Discovery learning pertains to the notion that students learn mathematics better if they are left to discover the rules and methods of mathematics for themselves, rather than being taught by teachers or textbooks (Mathematically Correct, n.d., para. 2). The approach encourages students to invent their own procedure for the arithmetical operations (Kamii et al, 1993). Instruction in “...which students construct meaning for the mathematical concepts and procedures they are investigating and engage in meaningful problem-solving activities” are considered ideal (Fuson, Carroll, & Drueck, 2000). According to Kamii (1991), the only way students can learn mathematics is by making their own decisions and evaluating the results of their decisions.

Whole Math. The premise behind whole math is to forego instruction in basic computational skills and emphasizes that instruction should focus on more complex skills (Kamii, 1991). It affirms that schools must eliminate all worksheets and replace memorization of algorithms. Basic computational skills are thought to make students into passive receivers of rules and procedures. NCTM stresses that practice should be purposeful and should focus on developing thinking strategies and a knowledge of number relationship rather than drill in isolated facts (NCTM, 2000, p. 82). NCTM’s recommendation aligns with the constructivists’ approach that educators should forego basic facts and introduce complex activities.

Logically it follows that Constructivists also emphasize the use of calculators. They argue that although basic computation skills are necessary in solving complex problems, calculators will compensate a student’s lack of component skills and allow the students to focus on problem solving rather than mere computation.

Cooperative Learning. The constructivist approach advocates learning through interaction and hence is a proponent of cooperative learning where students gather with peers in groups to construct strategies for solving mathematical problems, rather than sit in class with teachers instructing them (Cheney, 1997). Kamii et al. (1993) described an ideal classroom as one in which a teacher proposed a problem to a class and had them attain an answer through discussion. The teacher should not give them a hint or an answer until the class reaches an agreement. The authors argue that absolutely nothing is arbitrary in mathematical knowledge; therefore, the class will eventually reach the correct answer. NCTM recommends teachers to ensure that interesting problems and stimulating mathematical conversations are a part of each day (2000, p. 75).

Autonomy. Piaget (1973) stated that autonomy is the ultimate goal in education and refers to it as the ability to think for oneself. Kamii (1991) acknowledges Piaget's theory and stated that children can learn to make choices only by making their own decisions and evaluating the results of their decisions. Students are encouraged to make their own decisions and also to discuss with peers. They are provided with as much time as necessary for the critical thinking and discussion. Students are encouraged to not merely produce accurate answers but also to understand why a particular answer is correct. As is recognizable from their perspective, fluency in the performance of calculations is not valued. While most of the basic tenets of Constructivist theory differ from what Behavioral theory advocates, the extent to which fluency is valued is a major difference in the two views. In the next section, the behavioral approach to instruction is considered.

Behaviorist Approach

In order for the effective instruction to occur, one needs to understand how learning occurs for each student. The notion of how students learn best from a behavioral perspective generally follows a linear step by step progression as opposed to the more holistic view espoused by constructivists. Binder (2003), whose view typifies a behavioral perspective, suggested that it is helpful to view learning as occurring in three stages: initial learning for accuracy or quality, practice for fluency and endurance, and application or combination of the components into composite behavior. Binder's three stages of learning are similar to the learning hierarchy proposed by Haring, Lovitt, Eaton, & Hansen (1978). The learning hierarchy contains four stages: acquisition, fluency, generalization, and adaptation. With proper instruction, students first learn how to perform a skill accurately before they perform it fluently. The fluent performance of a skill, in turn, is believed to promote generalization to novel items, times, and settings by increasing potential performance capacity. Students will then be able to identify elements of previously learned skills that they can adapt to the new demands or situation.

In the following sections, the major hallmarks of a behavioral approach to instruction are elaborated upon. Embedded in each section below is an example of factoring equations in algebra using practices advocated by Johnson and Layng (1992), proponents of behavioral teaching methods.

Component Skills. A Number of researchers emphasize that component or basic skills first need to be taught in order for students to eventually develop more complex skills (Binder, 1993; Johnson & Layng, 1992, 1993; mathematically correct, n.d., para. 6). The students' task in solving a new problem is to first identify component activities

that would produce the desirable educational goal. Students can select those component activities based on the related and unrelated component performances that have been encouraged in the past (Johnson & Layng, 1992, 1993). In factoring equations in algebra, for example, component skills such as number writing, addition facts, isolating and solving for X in a simple linear equation, and squaring and factoring squared numbers are first introduced to the students (Johnson & Layng, 1992).

Fluency. According to the behaviorist approach, fluency is considered to be a necessary component for learners to achieve competency. Fluency is defined as the combination of accuracy plus speed and is typically converted to a rate-based metric such as number correct per minute (Binder, 1993, 2002, 2003). Binder (2003) remarked that a description of behavior without its temporal dimension is incomplete and ultimately false because it prevents researchers from seeing the performance beyond the attainment of 100% accuracy. Fluency has been shown to be an element in the development of “competency” within a wide variety of situations including special and general education (Shinn, 1989), adult literacy programs (Johnson and Layng, 1992), and customer call centers (Binder, 2002).

In addition to Binder’s (2003) definition of fluency, Johnson and Layng (1992, 1993) further affirm that fluency also requires the ability to quickly link a component behavior available in the environment with other component behaviors and to more complex behaviors. Every complex skill requires fluent component skills in order to perform those complex skills with competency. To be fluent, learners need to be able to identify related and unrelated component skills in their environment without hesitation.

To return to the example of factoring, once students are introduced to the component skills necessary to factor equations in algebra, the next step is instruction, which focuses on fluency building. To increase fluency, instruction generally incorporates practice with feedback. Johnson and Layng (1992) have suggested that unless component activities take place fluently, the higher-level comprehension will not occur.

Application. A student who is accurate and fluent in responding may still fail to apply component skills to new situations and settings. Another element necessary for a student to achieve competency is application. Application refers to students being able to use a skill in wide range of settings and situations, or to accurately discriminate between the target skill and similar skills.

It can also be said in solving factoring equations in algebra. Learners who have reached application level will not get confused whether to use addition, subtraction, multiplication, or division to solve the factoring. They may also be able to solve factoring equations from different workbook or outside of school.

Adaptation. Even when a student is able to use a skill in many situations, he or she may not yet be able to modify or adapt the skill to fit novel task-demands or situations. The goal of attaining competency is to become capable of identifying elements of previously learned skills that he or she can adapt to the new demands or situation.

Adaptation would be needed in factoring equations so that the student can adapt the skill to different demands or situation (e.g., calculus). When a student is presented with new environmental demands, the basic component behaviors he or she has learned

can apply and adapt to more complex and flexible activities (Johnson & Layng, 1992, 1993; Binder, 2002).

Review of Behavioral Research Pertaining to Academic Competency

In the preceding sections, behavioral and cognitive views have been outlined. Both theories are open to criticism. A strength of the constructivist view is “face validity” especially among school based professionals who tend to believe that students need to learn to think for themselves, and “drill and kill” associated with the behavioral approach is opposing to the development of “understanding”. However, while the roadmap to competency designed by the constructivists is more palatable to some, the outcomes have often been disappointing and the dismal mathematics performance of U.S. students is sometimes attributed to the constructivist approach (Clayton, 2000). The research base to support the constructivist approach has not evolved. The constructivists have often referred to basic research in cognitive psychology as supporting many of their underlying assumptions. However, this approach was criticized by three prominent cognitive psychologists including Noble Prize winner Herb Simon (Anderson, Reder & Simon, n.d., para. 1) who noted that “frequent misperception that the move from behaviorism to cognitivism implied an abandonment of the possibilities of decomposing knowledge into its elements for purposes of study and decontextualizing these elements for purposes of instruction.” (Anderson, et al., n.d., para. 1) They further suggested that that cognitivism, which is cited as a basis of constructivism, “does not imply outright rejection of decomposition and decontextualization.” (Anderson, et al, n.d., para. 1)

Behaviorists are most critical of the Constructivist approach because of its focus on accuracy as the component most essential for competency (Binder, 2003). The stage

of fluency building is skipped over and teachers use complex activities to teach component elements. Students devote themselves directly to solving complex problems and invent their own algorithms. However, this reverse manner of teaching is criticized by behaviorists as lacking the substantial element of fluency to advance students toward mastery (Binder, 1993). Behaviorists argue that by incorporating a measure of fluency, we gain knowledge on students' performance beyond 100 percent accuracy (Binder, 1993, 2003). With its percent-correct evaluation, accuracy does not allow us to detect the difference between instructional level and mastery level (Binder, 2003). Although it is a requisite component toward true mastery, accuracy is a poor predictor of whether performance will be retained, maintained, and applied to more complex situations (Johnson & Layng, 1992, 1993). Once a learner is able to perform a skill with speed and endurance, he or she is then ready to apply his or her skills to real-world context (Johnson & Layng, 1992, 1993). Studies in support of these general assertions by behaviorists are reviewed below.

Studies conducted by behaviorists focus on identifying the importance of fluent component skills to be able to perform composite skills. In general education, as students progress toward intermediate grades, they are expected to perform more complex activities in each subject, which will be difficult for students who are performing at frustrational level. The purpose of each subject changes to remembering and understanding key concepts, integrating new material with prior knowledge, and applying the knowledge to problem solving (MacArthur & Haynes, 1995). Although the studies introduced in the following section focused on reading, the behavioral principles on how a student achieve competency are similar to that on mathematics.

CBM Validity Studies in Reading. This section reviews literature pertaining to Curriculum-based Measurement as it applies to reading. Although the focus of the present study is on mathematics, research pertaining to reading is more comprehensive. Because of the emphasis on fluency in reading, some of the results of studies on reading have applicability to mathematics. In the area of reading children who have achieved competency, decoding is a highly automatic task. If decoding consumes too much performance capacity, then the extra effort taken in decoding will detract from comprehension at sentence, paragraph, and text levels (Tan & Nicholson, 1997). Increases in the speed of performance improves the range of a student's potential performance capacity, enabling application or combination of skills into more complex behavior (Binder, 1993; Helwig et al, 1999)

One prominent method of assessing fluency is known as Curriculum-based Measurement (CBM). CBM is a standardized set of measurement procedures that can be used to evaluate performance outcomes in the basic academic skill areas of reading, spelling, mathematics, and written expression (Shinn, 1989). CBM can summarize and inform decision making for use in screening, eligibility determination, instructional planning, and program evaluation (Deno, 1986). In the area of reading, the number of words a student reads correctly from a reading passage in a one-minute interval is counted. For mathematics, the number of correctly written digits during a two-minute interval from grade-level computational problems is counted (Shinn, 1989). Deno, Mirkin, & Chiang (1982) conducted a CBM validity study in reading using CBM and a number of criterion-referenced tests. The criterion-referenced tests utilized in the study were the Stanford Diagnostic Reading Test, the Woodcock Reading Mastery Test, and

the Reading Comprehension subtest from the Peabody Individual Achievement Test. The researchers found that number of words read per minute correlated highly with the results from the norm-referenced tests. Most of the correlation coefficients were above .80. The researchers therefore concluded that passage reading was a valid measure of students' reading skill.

Burns et al. (2002) investigated the relationship between reading fluency and comprehension by having third- and fourth-grade students as participants. The researchers utilized four norm-referenced reading tests in which students read passages with scrambled words orally and answer inferential and literal comprehension questions after completing each passage. The passages included an incremental percentage (0%, 10%, 20% and 30%) of scrambled words. The percentage of comprehension questions answered correctly as well as the reading rates served as the dependent measures. Results demonstrated that the incremental increase in scrambled words significantly reduced reading fluency and the resulting comprehension, providing with evidence of the direct link between reading comprehension and fluency. Faster readers tend to have higher comprehension and make more inferences from written materials than do students reading at lower rates (Engen & Høien, 2002). There have been ample additional studies that examined the relationship between fluent component skills and comprehension in reading using CBM (e.g., Marston, 1989; Shinn, Good, Knutson, Tilly & Collins, 1992; Kuhn & Stahl, 2003). Although correlational studies provide practical information to a great extent, it does not reveal the causal relationship. The studies introduced below are several interventions utilizing fluency in reading to strengthen students' reading comprehension.

Fluency-building Interventions on Reading. Graham, Harris, and Chorzempa (2002) examined whether supplemental spelling instruction had an effect on spelling, writing, and reading for second-grade students who were experiencing difficulty learning to spell. In addition to regular spelling class, the experimental group was provided with supplemental spelling instruction focused on phonemic awareness and fluency. The supplemental instruction included teaching the students common sound-letter combinations, spelling patterns or rules involving long and short vowels, and frequently occurring phonograms or rhymes. Students in the control condition were taught mathematics skills instead. The instructors taught the skills using word sorting and timed how long it took the students to complete the task. The students received reinforcement if they sorted correctly and exceeded their previous scores. The authors found a statistically significant difference between the experimental and control conditions in all spelling, writing, and reading post assessments. Results demonstrated that building fluency on component behavior (spelling) enables one to excel their performance on composite behavior (spelling, reading, and writing). Inefficient phonological awareness processes will take an excessive share of mental resources available for comprehension and will produce less efficient comprehension. There are studies that have likewise demonstrated the impact of phonological awareness on reading comprehension (Connerly, Johnston, & Thompson, 2001; Engen & Høien, 2002).

Tan and Nicholson (1997) investigated the relationship between fluency and comprehension by comparing school-age children who have received training on rapid decoding to those who never have received the training. The training facilitated flashcards, and the condition varied in the single-word training condition, the phrase-

training condition, and the control condition. In the single-word condition, students were taught to recognize each target word by using flashcards. In the phrase-training condition, students were shown some sentence cards as well as phrase cards, containing the target words from the passage in which they will read after the training. Practice in reading the flashcards continued throughout the 20-minute training session, with the aim of achieving the preset criterion rate of 90 words per minute. Trained and untrained students read aloud passages containing target words after the training, answer comprehension questions, and retell the passage in their own words. Results showed that both of the trained conditions were superior to the control condition on comprehension scores. As is apparent from the results of the studies above, fluency appears to be a vital component for students to achieve comprehension in area of reading.

The results of the reading studies reviewed here illustrate that students who are fluent on component skills have better reading comprehension. In the next section, studies pertaining to mathematics are reviewed.

CBM and Mathematics. Although CBM is a well-established technology for measuring student proficiency in reading, less is known about the technical adequacy of CBM in mathematics. Shinn and Marston (1985) conducted a study to obtain construct validity evidence for CBM mathematics measures and found that CBM mathematics probes differentiated students in general education from students with mild disabilities and also from general education students in different grade. Thurber, Shinn, and Smolkowski (2002) investigated the relation of CBM in mathematics to the constructs of general mathematics achievement, computation, and application using confirmatory factor analysis. Results indicated that computation and applications were distinct, but

highly related constructs. The researchers also suggested that skills in one area are necessary for success in the other.

The link between fluency in basic computation skills and higher order mathematics was noted by Gardner et al. (2001) who investigated the effectiveness of an after-school program on elementary school aged students' reading and mathematical performances. In this study, ten urban at-risk African-American male students participated during the course of one year. The students were one or more years below grade level in both reading and mathematics. The mathematics intervention included a reciprocal peer-tutoring program incorporating flashcards. At the beginning of a week, all students were administered a timed test to establish a baseline. During the week, the tutor and tutee changed roles during sessions lasting for two-minutes. At the end of each week, an adult experimenter reviewed the multiplication flashcards with each student. A multiplication card was considered to be mastered if a student could respond correctly to the question presented in less than three seconds. Results showed that the use of the peer-mediated interventions improved both the reading and mathematics skills of students. In mathematics in particular, every student improved both his or her accuracy and fluency rates on multiplication facts. The group's mean accuracy difference score was 52.5% across all math facts and the mean fluency rate was 35.2% across all multiplication facts. Of particular interest to the present investigation, the author also noted that the students who could not respond accurately and fluently to basic multiplication facts had difficulty with division, fractions, percentages, and algebra items on the state's mathematics proficiency test.

The purpose of the present study was to further examine the linkage between component skill fluency and overall mathematical competence in children. The impetus for the study derives from reading where it has been shown that Reading Fluency is highly correlated with Reading Comprehension. The purpose of this study is to investigate whether mathematics fluency in component skills correlates with mathematics “comprehension.” Johnson and Layng (1992) affirm, “...it is only the accumulation of weak component skills that makes learning harder and harder”. If a student is fluent in the basic component skills necessary in mathematics, the student would be expected to perform better on more complex materials in mathematics than students who were not fluent in the component skills.

It should be noted that Comprehension is a term used in reading and has not been used in the literature in connection with mathematics. Still there is a recognition in mathematics that skill in one area of mathematics is linked to and facilitates understanding in other areas. These linkages have been discussed in both the constructivist and behavioral literature. In the constructivist literature, there is a tendency to use terms such as “Number sense” (Kamii, 1985). In the behavioral literature, Harniss, Stein, and Carnine (2002) used the term “big ideas” which are viewed as overarching instructional goals that imply a deeper understanding of mathematics. Kame’enui and Carmine (1998) define Big Ideas as “...major organizing principles that have rich explanatory and predictive power and are applicable in many situations and contexts”. Seven components comprise the big ideas: Place value, Expanded notation, Commutative property, Associative property, Distributive property, Equivalence, and The “rate of composition/decomposition of numbers.” These foundational ideas enable students to

apply composite behaviors. When teachers select learning goals for their students, it has been recommended that the goals be focused on big ideas (Harniss, Stein, & Carnine, 2002) because of their more general applicability across mathematics. It is assumed that once learners become fluent in big ideas, they can gain foundational knowledge that will be useful for understanding more complex operations.

Current Investigation

In the present study, one goal was to examine the relationship between component skills and mathematical “comprehension”. Hence, there was a need to operationalize and measure the “comprehension”. In the present study, comprehension was defined as performance on a broad based test of mathematics. The purpose of the test used in the present study was not to measure specific skills, but to provide an index of global mathematics proficiency, similar to that obtained from an omnibus achievement test. The current study utilized a criterion-reference test as a measure of the students’ mathematical comprehension. The criterion-related validity of fluency on specific skills as measured by CBM probes was evaluated in the present study by examining the extent to which the CBM measures corresponded to other commonly accepted indexes of mathematical proficiency. That is, concurrent validity estimates were obtained of formative measures using traditional criterion-referenced test performance as the criterion (Deno, 1985). High correlation between the achievement test and the CBM measures would serve as evidence that the component activity assessed in CBM measures satisfy standards for concurrent validity and can be used as an index of general mathematical proficiency. In addition, by including an omnibus test of mathematical achievement and examining the relationship of the various measures used in this study to this more global measure of

“mathematical comprehension” (i.e., the relationship between the omnibus mathematics achievement tests and other measures of mathematical skill), the present study was intended to shed light on which of the brief measures would be associated with general mathematics competence.

METHOD

Participants and Setting

A total of 140 students in second- through fourth-grade classrooms from an elementary school located in the Southeast participated in the study. Students from general education were considered for participation. All phases of the experiment took place in the students' regular classrooms under the supervision of the experimenter or an assistant.

Materials

Each student participating in the study was assessed using three Curriculum-Based Measurement (CBM) probes (i.e., single- and multiple-skill mathematics computational probes and Maze reading passages), the mathematical reasoning probe, the Big Ideas probe and one criterion-referenced assessment (i.e., Stanford Diagnostic Mathematics Test, fourth edition). In addition, teacher perceptions of the curriculum-based measurement probes was obtained. A brief description of each type of measurement follows.

Curriculum-Based Measurement: Single-skill Mathematic Computation Probe (SSMCP). The single-skill mathematic computation probe consisted of computational problems of one basic mathematics operation (i.e., addition, subtraction, multiplication, or division). Starting from a list of skills derived from their districts' curriculum standards, teachers within a specific grade identified a specific computational skill for which instruction has recently been completed.

The computation probes were generated utilizing the Mathematics Worksheet Factory Deluxe V3 © (1998-2002). The software was developed by Schoolhouse Technology Corporation to facilitate the creation of mathematics materials. Users can

select problem sets from eight categories; concepts, operations, fractions, percent, measurement, geometry, graphing or algebra. The program allows users to select the number of problems per page; how the problems are to be displayed; and the number of digits, decimals, multiples, addends, or remainders to be used for each problem. A trial version of the software is available from

<http://www.schoolhousetechnologies.com/products/download.htm>. Based on the study conducted by Hintze et al. (2002), the two-minute single-skill mathematics computational probe was administered only once. For the current study, the probes contained 40 questions, and each problem was presented vertically.

Curriculum Based Measurement: Multiple-skill Mathematics Computation

Probes (MSMCP). The multiple-skill mathematics computation probes consisted of a sample of basic math operation (i.e., addition, subtraction, multiplication, and/or division), which are grade appropriate according to the state curriculum guide. The computational skills included in the probes were selected from a list of skills nominated by teachers within each grade. The probe was generated from the same software described above, in the same manner as the single-skill mathematical computational probe was generated. The probes contained either 40 or 42 questions with an equal number of each operation (e.g., 20 problems in addition and 20 problems in subtraction for second grade probe). Based on a previous study, the students will be given three equivalent MSMCPs (Hintze et al., 2002).

Mathematical Reasoning Probe. A mathematical reasoning probe was constructed using items that reflect constructs described by the NCTM Standards. The purpose of this probe was to evaluate the validity of assessments designed to evaluate

students' mathematical thinking and reasoning skills. There are ten NCTM Standards that apply across each grade-level, and several specific content areas are emphasized in each grade band. For the grade band of pre-kindergarten through grade 2, the core program areas are the Number and Operations and Geometry (see Appendix A for a definition of each content area). For the grade-band of grades 3-5, Multiplicative Reasoning and Equivalence (Algebra) are described as the central themes (see Appendix B for a definition of each theme). The definitions from each content area are available from <http://standards.nctm.org/document/chapter1/index.htm>. Among the content areas emphasized as essential by the NCTM, an area in which the definition matched with one of the six content areas of the state's curriculum standard was selected. In order to be consistent with the curriculum of the district in which the study was conducted, the mathematics text and an accompanying workbook used by the district were then utilized as an item pool from which to draw items. That is, the type of items selected was guided by NCTM standards but the actual items were taken from curricular materials used in the district. The mathematical reasoning probes for each grade contained 20 items.

Big Ideas Probe. In an attempt to evaluate the validity of assessments designed to review students' knowledge of the major organizing principles, a big ideas probe was constructed. The probe was generated using items that reflect organizing principles described by Harniss, Stein, and Carnine (2002). There are seven organizing principles in big ideas that are considered to have rich explanatory and predictive power and are applicable in many situations and contexts (Kame'enui & Carnine, 1998). Amongst the big ideas principles highlighted as crucial by Harniss, Stein, and Carnine (2002), the

principle of place value was selected for two reasons. First, this skill is more easily operationalized and measured than others. Second, place value is an important skill at all elementary grade levels as evidenced by its prominence on the state's curriculum standards for each grade. The big ideas probes for each grade contained 20 items.

Curriculum Based Measurement: Maze Reading Passages. Studies have demonstrated the importance of students' reading skills in performing mathematics effectively (Clements, 1980; Clarkson, 1983; Helwig et al., 1999; Helwig et al., 2000). In order to assess the contribution of reading to overall mathematics competency, Maze reading probes were administered. Since the primary focus of the study was mathematics, Maze probes were chosen as a fast but valid method of assessing reading. Studies that have examined the validity of maze reading have indicated they have acceptable psychometric properties. Jenkins and Jewell (1993) investigated whether Maze was a suitable measure of reading performance and reported correlations higher than .80 both with the Metropolitan Achievement Test (MAT) ($r = .80, p < .01$) and Gates-MacGinitie Reading Test ($r = .85, p < .01$). Fuchs and Fuchs (1990) examined the relation between performance on second- to third- grade maze passages and the Reading Comprehension subtest of the Stanford Achievement Test. The average correlation between the two assessments was .77. In addition to its efficiency, Maze is claimed to assess reading proficiency including an explicit comprehension element and therefore has better face validity than Oral Reading Fluency (Jenkins & Jewell, 1993).

The probes for the second through the fourth grades were constructed from the bank of reading passages created by Louisiana State University School Psychology Department. Information about the reading passages is available from

<http://bitwww1.psyc.lsu.edu/reading%20center.htm>. Once the original reading probes were selected, the Maze passages were generated by deleting every seventh word after the first sentence as has been recommended by Fuchs and Fuchs (1992). In place of the deleted word, a blank was inserted. Next to the blank, in each case, three alternative words were printed including one correct word and two incorrect words. Based on the study conducted by Fuchs, Fuchs, and Deno (1982), students were administered three two-minute Maze probes during one complete assessment session, in order to control the varying degrees of difficulty of the reading passages. The score used in most of the analyses was the median score of the three passages administered.

The Readability of the passages were calculated by Readability Calculations v.3.7 © (2000) drawn from Micro Power and Light Company. The Spache readability formula, widely used in assessing first through fourth grade, was utilized in the current study. Passages were evaluated prior to formatting for the maze and were at the student's grade placement level.

Stanford Diagnostic Mathematics Test, Fourth Edition (SDMT-4). The SDMT-4 is a widely used group administered assessment containing six levels ranging from Red (for grades 1 to 2) to Blue (grades 9 to 13). Students in the current study consisted of second, third and fourth graders. They were given the Orange, Green and Purple levels, respectively. At each level, there are 32 multiple-choice questions pertaining to Concepts and Applications, and 20 multiple-choice questions pertaining to Computation. The Computation area includes items on algorithms of addition, subtraction, multiplication, and division. The Concepts and Applications area measures

number systems and numeration, patterns and functions, graphs and tables, problem solving and geometry and measurement.

Internal consistency reliability estimates generally are above .80 for the SDMT-4. Evidence of construct validity is found in the correlations obtained between the SDMT-4 and the Otis-Lennon School Ability Test, Sixth Edition. The correlations among subtests on the two instruments are in the .60s and .70s (Impara & Plake, 1998).

Teacher Perceptions of the Current Probes. In an attempt to obtain information on social validity, teachers were asked to express their satisfaction with the mathematics curriculum-based measurement probes and the mathematical reasoning probe. The questionnaire along with an example of each probe was handed to the teachers on the first day of the administration and was collected the following day. The questionnaires were collected before the probes had been scored. Teachers were asked to respond to the following statements: 1) Probe X will accurately measure student performance in mathematics, 2) Probe X will reflect the strengths and the weaknesses the student have, and 3) I would use Probe X for assessing student performance in mathematics. Teachers will respond by circling one of the corresponding numbers presented in standard Likert scale format (i.e. 1= Strongly agree, 2= Agree, 3= Neutral, 4= Disagree, 5= Strongly disagree). These items derive loosely from similar scales used in treatment acceptability research (Witt & Martens, 1983) but were constructed by the author for this study. There are no data on the reliability or validity of this scale.

Dependent Measures

For the single-skill mathematics computation probe and the multiple-skill mathematics computation probes, the number of correctly written digits during a two-

minute interval served as the measures of students' performance (Shinn, 1989). Both probes were scored in terms of the number of correctly written digits following methods described by Shinn (1998). For the multiple-skill mathematics computation probes and the maze reading passages, the median score of the three probes were used as the student's score. For the mathematical reasoning probes and the big ideas probe, the number of items answered correctly served as the measures of students' performance. For the CBM Maze reading passages, the number of correctly selected words was counted as the measure (Shinn, 1998). The SDMT-4 was hand-scored by a trained school psychology doctoral student. The total score of the Concepts and Application area and the total score of the Computation area were used from the SDMT-4.

Procedures

This study was completed over approximately a two-week period. Three CBM probes, the mathematical reasoning probe, the Big Ideas probe and a criterion-referenced assessment were administered to each student. Every assessment was group-administered in each participating classroom. Before the administration of the first assessment, the current experimenters handed the teacher a copy of the social validity questionnaire and asked that it be completed while the students were being assessed. The first assessment in all cases was the Stanford Diagnostic Mathematics Test, fourth edition. Three school psychology graduate students administered the SDMT-4, according to the procedures described in the administration manual. The remainder of the five probes was administered in a randomized order in an attempt to avoid order effects. Either two to three probes were administered in one day with the reminder being administered on the following day for each class. Procedures similar to those described

by Shinn (1989) guided the administration of the SSMCP (Appendix C). The administrative procedure for the MSMCPs was identical to that of the SSMCP. Two minutes were provided to complete each probe.

The mathematical reasoning probe and the Big Ideas probe were administered in a similar manner as the two previous math assessments with one exception. That is, the students were allowed to complete the probes with no time limit. The students turned their probe over when completed. The Maze reading passages were administered in accordance with procedures described by Shinn (1998). Two minutes were provided to complete each probe, and the probes were collected after every assessment. Each one of the Maze probes was administered separately at one testing session. Finally, the teachers turned in the completed rating of the assessments prior to seeing the results.

Interrater Agreement

Interrater agreement were determined by having two independent persons score approximately 20% of each of the assessments used including the math CBM assessments, mathematics reasoning probe, the Big Ideas probe, maze reading passages and the SDMT-4. Interrater reliability was calculated as the percentage agreement for individual items on math and reading probes, and was calculated by counting the number of agreements and number of disagreements for all attempted items, dividing the number of agreements by the number of agreements plus disagreements, and multiplying the resulting number by 100%. Average interrater agreement for the assessments was 99.65% (99.15-100%).

DATA COLLECTION AND ANALYSIS

To investigate the effect of fluency on component behavior on mathematics achievement, four separate analyses were conducted. Pearson product-moment correlation coefficients were calculated between three alternative forms of multiple-skill mathematics computation probes and three alternative forms of Maze reading passages in order to examine alternate-form reliability of the probes. Subsequently, bi-variate correlation coefficients were calculated between the criterion variable (i.e. Stanford Diagnostic Mathematics Test, Fourth Edition) and multiple predictor variables (SSMCP, MSMCP, Mathematical Reasoning Probe, Big Ideas Probe and Maze). Using the Bonferroni control for increased Type I errors with six variables, a p-value of less than .008 ($.05/6 = .008$) was required for significance. Following the correlational analyses, a test of significance of the difference between dependent correlation coefficients was conducted for each grade to evaluate if the correlation coefficients between particular predictor variables and the SDMT- 4 are significantly higher or lower than other predictor variables. Multiple regression analysis was used to identify which predictor variables most effectively predicted student's mathematical comprehension. Median scores were being included for the MSMCP and the Maze probes during each analysis.

However, in order to acquire a compelling result, multiple regression analyses require a minimum of 40 participants per predictor variable (Cohen & Cohen, 1983; Tabachnick & Fidell, 1996). There were six variables total, and 140 students participated in the study. Not having enough participants to run the analysis at once, the predictor variables were eliminated by running several single-regression analyses. First, as both are representative of basic computational tasks, an analysis between the SSMCP and the

MSMCP median was run to identify the stronger predictor. Subsequently, an analysis between the mathematical reasoning probe and the Big Ideas probe was run. Each probe represented non-computational tasks from different approaches. Results of the two single regression analyses indicated that the MSMCP and the Big Ideas probes were the stronger predictor variables in relation to the competing variables. Finally, the scores from the Maze probes were included in the multiple regression equation as one variable because the Maze was the single variable that measured students' reading performance.

RESULTS

Alternate-form Reliability of the Probes

The purpose of the first set of analyses was to examine the alternate-form reliability of the three mathematics computation probes and three Maze reading passages. In order to accomplish this, Pearson product-moment correlation coefficients were calculated.

The alternate-form reliability between MSMCP 1 and MSMCP 2, MSMCP2 and MSMCP 3, and MSMCP 1 and MSMCP 3 equaled .771, .794, and .805, respectively. Likewise, the alternate-form reliability between Maze 1 and Maze 2, Maze 2 and Maze 3, and Maze 1 and Maze 3 were .762, .835, and .663, respectively. Each of these correlation coefficients was significant at the .01 level for a two-tailed test. Given these results, the median scores for both MSMCPs and Mazes were used in subsequent bi-variate correlation and multiple regression analyses.

Criterion-related Validity

The purpose of the second set of analyses was to investigate the criterion-related validity of the various predictor variables of interest in the study (i.e., SSMCP, MSMCP median, Mathematical Reasoning Probe, the Big Ideas probe and Maze median) with respect to the criterion variable (i.e., SDMT-4). The correlation coefficients between the SDMT-4 and the predictor variables are presented in Table 1. Most of the correlation coefficients were significant at the .008 level. Consistent with previous studies (Clements, 1980; Clarkson, 1983; Helwig et al., 1999; Helwig et al., 2000), Maze reading passages correlated moderately with the SDMT. ($r = .522$, $p < .008$).

Table 1

Correlations Between the SDMT- 4 and the predictor variables

Grade	SSMCP	MSMCP	Math reasoning	Big Ideas	Maze
2	.645***	.609***	.348*	.532***	.681***
3	.332	.487**	.601***	.845***	.621***
4	.648***	.590***	.605***	.606***	.619***
N	.500***	.546***	.474***	.507***	.522***

* significant at the .05 level. ** significant at the .01 level. *** significant at the .008 level under Bonferroni correction.

Further, the predictor variables were rank ordered, and a test of significance of the difference between dependent correlation coefficients (Glass & Stanley, 1970) was conducted. For the second grade, the analysis revealed that the correlation coefficient for the SSMCP ($r = .645$) was significantly higher than the correlation coefficient for the mathematical reasoning probe ($r = .348$, $t(43) = 2.51$, $p < .01$). The same significant difference was found between the MSMCP ($r = .609$) and the mathematical reasoning probe ($t(43) = 1.92$, $p < .05$). There was no significant difference between other variables. For the third grade, a significant difference was found between the Big Ideas probe and every other probe. That is, the correlation coefficient for the Big Ideas probe ($r = .845$) was significantly higher than the correlation coefficient for the maze ($r = .621$, $t(26) = 2.03$, $p < .05$), mathematical reasoning probe ($r = .601$, $t(26) = 2.28$, $p < .05$), MSMCP ($r = .487$, $t(26) = 2.55$, $p < .05$), and SSMCP ($r = .332$, $t(26) = .560$, $p < .01$). No significant difference was found between other variables. For the fourth grade and all grades total, however, no significant difference between the correlation coefficients was found.

In order to determine how much of the variance in the SDMT- 4 could be explained by the predictor variables, in the next set of analyses, multiple regression was used. Results of stepwise regression using the MSMCP, the Big Ideas probe, and the Maze reading passage score as independent variables showed that the MSMCP, entered first in the equation and accounted for 54.7 percent of the total variance in the SDMT- 4. The Big Ideas probe was entered second into the equation and, together with the MSMCP, accounted for 64.4 percent of the variance. Entering the Maze reading passage score as the third variable, the predictor variables accounted for 66.1 percent of the

variance in the SDMT- 4 for all of the participants in the study. No further variables entered the equation.

Results of the teacher perceptions of the probes showed that the teachers were satisfied with the probes. Mean rating for the first, second and third question was 2.17, 2.08 and 2.42, respectively.

DISCUSSION

The purpose of the study was to investigate whether fluency in component skills was associated with overall proficiency or “comprehension” in mathematics. The current study attempted to measure mathematical comprehension using the SDMT-4. It was hypothesized that students who were more successful (i.e., fluent) in their performance on mathematical component skills would also have better comprehension of more difficult mathematical concepts and would, therefore, score well on a general mathematics achievement test. It was further anticipated that students’ measures of fluency on component skills would have a stronger relationship with an omnibus math achievement test than would student performance on a test of mathematical reasoning derived from constructivist theory. In order to examine these relationships, bi-variate correlation coefficients and multiple regression analyses were conducted.

The results indicated that those students who performed well on component activities (SSMCP and MSMCP) also performed well on the comprehensive proficiency activities (i.e. SDMT-4). Correlation coefficients between the SDMT- 4 and the mathematics reasoning probe while statistically significant were not as high as between the component activities (SSMCP and MSMCP) and the SDMT- 4 in the second grade. Interestingly, in third grade, the big ideas probe had the strongest association with the SDMT- 4. This correlation was significantly higher than for the other independent measures. Consistent with previous studies (Clements, 1980; Clarkson, 1983; Helwig et al., 1999; Helwig et al., 2000), Maze reading passages correlated moderately with the SDMT- 4, adding further evidence that a reading skill is highly related to students performing some higher order mathematics problems.

The current study is supportive of other studies such as that of Thurber, Shinn, and Smolkowski's (2002) who found promising evidence for the validity of CBM in predicting more global mathematical assessments. Johnson and Layng (1992) affirm that component skills and complex skills are highly related. They asserted that it is the accumulation of weak component skills that makes learning more challenging. A number of validity studies in the area of reading led researchers to conclude that passage reading was a valid measure of a student's reading skill. This has not been accomplished in the area of mathematics; however, the current study provides an additional step in establishing the relationship.

The Big Ideas probe was a strong predictor of students' overall proficiency in mathematics, especially in the third grade. It appears puzzling at first why behaviorists such as Harniss, Stein, and Carnine (2002) emphasize "big ideas" when other behaviorists stress the importance of component skills. Researchers promoting big ideas affirm that educators must select goals that address important concepts and skills, and an analysis based on big ideas reduces the number of formulas students must learn (Harniss, Stein, & Carnine, 2002). A student, who becomes fluent in component skills or big ideas, when presented with a new environmental requirement, can recombine the component skills in new ways that correspond to the higher level complex skills. Researchers stressed that increasing the speed of performance improves the range of a student's potential performance capacity, enabling them to put greater effort in solving higher order activities (Johnson & Layng, 1992, 1993; Binder, 1993; Helwig et al., 1999). Conversely, if a student is not fluent in those tasks, less performance capacity is available for him or her to perform activities that are more complex and, therefore, the overall

performance of the student would be poor. For example, in reading, if a task consumes too much performance capacity, the extra effort taken in the component skills will detract from comprehension at sentence, paragraph, and text levels (Tan & Nicholson, 1997). Big ideas and component skills are therefore highly related and are both consistent with a behavioral approach.

Although the current study was successful and provided a starting point for future research, several limitations should be noted. Initially, although the standardized assessment (i.e., SDMT- 4) used in the study was adequate for the purpose of the research, it may be beneficial for future studies to use other standardized assessments as well in order to further explicate the relationship between fluency and comprehension in mathematics. Additionally, the mathematical reasoning probe, which was generated using items that reflect a constructivist's view, and the Big Ideas probe administered in the current study could have been improved to better measure what truly represented their approach by including every content area and the organizing principle in the probe. Another limitation concerning generating mathematical reasoning probe is the possible difficulty in replicating the construction of the probe. Furthermore, a sample size large enough for multiple regression analyses of each grade would have allowed detailed analyses within each grade.

Future researchers may wish to examine whether an intervention to enhance component skills may increase the mathematical comprehension. The present study is merely correlational. Although correlational studies provide practical information, they do not allow causal interpretations. Demonstrating the effectiveness of a fluency building intervention in component skills in strengthening students' mathematics

comprehension would unveil the causal relationship between fluency and comprehension in mathematics. If a causal relationship could be established between the two, this may show the beneficial effect of intervention on mathematics comprehension for students performing poorly in the mathematics component skills. If the benefits of fluency training can be documented experimentally, this may lead to greater use by educators (Bucklin, Dickinson, and Brethower, 2000).

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APPENDIX A

DEFINITIONS OF MATHEMATICAL CONTENT AREAS CRUCIAL FOR GRADE 2

Number and Operations. Understand numbers, ways of representing numbers, relationships among numbers, and number systems. Understand meanings of operations and how they relate to one another. Compute fluently and make reasonable estimates.

Geometry. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Specify locations and describe spatial relationships using coordinate geometry and other representational systems. Apply transformations and use symmetry to analyze mathematical situations. Use visualization, spatial reasoning, and geometric modeling to solve problems.

APPENDIX B

DEFINITIONS OF MATHEMATICAL THEMES CRUCIAL FOR GRADE 3-5

Multiplicative thinking. Develops knowledge that students build on as they move into the middle grades, where the emphasis is on proportional reasoning.

Algebra /Equivalence. Understand patterns, relations, and functions. Represent and analyze mathematical situations and structures using algebraic symbols. Use mathematical models to represent and understand quantitative relationships. Analyze change in various contexts.

APPENDIX C

ADMINISTRATION PROCEDURES FOR MATH CURRICULUM-BASED MEASUREMENTS

- 1) Note to the class whether single-skill or multiple-skill mathematics computation probes are to be administered.
- 2) Say to the students: “This is a math quiz.”
- 3) For single-skill probes say: “All of the problems are [addition, subtraction, multiplication or division] facts.”

For multiple-skill probes say: “There are several types of problems on the sheet. Some are addition, some are subtraction, some are multiplication, and some are division [as appropriate]. Look at each problem carefully before you answer it.”
- 4) “When I say ‘start’, turn them over and begin answering the problems. Start on the first problem on the left on the top row [point]. Work across and then go to the next row. If you can’t answer the problem make an ‘X’ on it and to the next one. When I tell you to begin, turn the sheet over and start answering the problems. When I tell you to stop, put your pencil down. You will have two minutes to complete as many problems as you can. Are there any questions?”
- 5) Say “Start.”
- 6) Monitor student performance so that students work the problems in rows and do not skip around or answer only the easy problems.
- 7) After 2 minutes, say “Stop”, and start collecting the probes.

VITA

Chisato Komatsu received a Bachelor of Art degree at the University of Texas at Austin (UT Austin) in 2001. Her undergraduate experience was a liberal arts education with an emphasis in psychology. During her years at UT Austin, Chisato received various awards and honors including The University of Texas President's Honor Role. She has also served in various leadership roles in the International Student Association as a Secretary, in a Dorm Council as a Secretary, and in Texas Student Psychological Association as a Treasurer. In her junior and the senior years at UT Austin, she enrolled in a class and two experimental laboratories with Dr. David Buss that focused on evolutionary psychology. Through the experience, Chisato discovered an interest in areas of psychology that takes evolutionary theory into its approach. Together with the experience she has gained at UT Austin and her long time interest in working with children with behavioral and academic problems, Chisato discovered she would like to dedicate her career to the study of behavior. Thus, she is currently pursuing her doctoral degree in school psychology at the Louisiana State University.